**Problem Set 1** **Due Date:** February 13th, 2026

1. Consider a standard deck of 52 cards: there are 4 suits, each with 13 consecutive values. A hand consists of a set of 5 cards.
   (a) How many hands with all cards in the same suit are there?
   (b) How many hands with all consecutive cards are there? Here, consecutive does not loop around (e.g., the hand with cards $\{10, 11, 12, 13, 1\}$ is not consecutive).
   (c) Which of the former two types of hands is more likely for a random hand?
2. Show that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ for any $n \geq 1$.
3. How many lists of $m$ 1s and $n$ 0s with exactly $k$ consecutive runs of 1s are there? Here, a run is a maximal set of consecutive entries with the same value. Note that:
   - $\left(\binom{n}{k}\right)$, i.e. $n$ multichoose $k$, denotes the number of ways of choosing $k$ elements, with repetition, from a set of size $n$.
   - $\left(\binom{n}{k}\right) = \binom{n+k-1}{k}$ is a known identity.
4. Let $G = (V, E)$ be a connected graph with pairwise distinct costs $c : E \to \mathbb{R}_{>0}$. Let $C \subseteq E$ by any cycle in $G$ and let $e = \arg\max_{e \in C} c_e$. Show that $e$ does not belong to any minimum spanning tree of $G$.