Problem Set 4 **Due Date:** May 8, 2026

1. A maximal outerplanar graph is a (simple) outerplanar graph that is not a proper subset of another outerplanar graph on the same nodes. Use Euler's Formula to prove that an n-node maximal outerplanar graph has $2n - 3$ edges.

2. Prove that every outerplanar graph can be properly 3-colored.

3. A maximal outerplanar graph is a (simple) outerplanar graph that is not a proper subset of another outerplanar graph on the same nodes. Prove that the node set of a maximal outerplanar graph can be partitioned into two sets, one of which induces a tree and one of which induces an independent set.

4. Let $K_{m,n}$ be the complete bipartite graph with m nodes on the left and n nodes on the right. Show that
$$|\mathrm{ST}(K_{n,m})| = m^{n-1}n^{m-1}.$$